Final Jeopardy!

Appendix	Ch. 1	Ch. 2	Ch. 3	Ch. 4	Ch. 5
200	200	200	200	200	200
400	400	400	400	400	400
600	600	600	600	600	600
800	800	800	800	800	800
1000	1000	1000	1000	1000	1000

Appendix 200

Is the triangle with side lengths 17, 15, and 8 a right triangle? Why/Why not?

Appendix 200

If a right triangle, the Pythagorean theorem should hold:

$$c^2 = a^2 + b^2$$

$$17^2 = 15^2 + 8^2$$

$$289 = 225 + 64$$

$$289 = 289$$

Yes it is a right triangle

Appendix 400

Find the quotient and remainder

$$\frac{(1 - x^2 + x^4)}{(x^2 + x + 1)}$$

Appendix 400

$$\frac{(1-x^2+x^4)}{(x^2+x+1)}=$$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x^2+x+1 & x^4+0x^3-x^2+0x+1 \\ & \underline{x^4+x^3+x^2} \\ & -x^3-2x^2+0x+1 \\ & \underline{-x^3-x^2-x} \\ & x^2+x+1 \\ & \underline{x^2+x+1} \\ & 0 \end{array}$$

$$\therefore \frac{(1-x^2+x^4)}{(x^2+x+1)}=x^2-x+1$$

Appendix 600

Solve for *x*

$$\frac{4(x-2)}{x-3}+\frac{3}{x}=\frac{-3}{x(x-3)}$$

Appendix 600

$$\frac{4(x-2)}{x-3}+\frac{3}{x}=\frac{-3}{x(x-3)} \Rightarrow x \neq 0,3$$

$$\frac{x}{x}\frac{4(x-2)}{x-3}+\frac{x-3}{x-3}\frac{3}{x}=\frac{-3}{x(x-3)}$$

$$4x(x-2)+3(x-3)=-3$$

$$4x^2-8x+3x-9=-3$$

$$4x^2-5x-6=0$$

$$x=\frac{5\pm\sqrt{5^2-4(4)(-6)}}{2(4)}=\frac{5\pm\sqrt{121}}{8}=\frac{5\pm 11}{8}=2,-\frac{3}{4}$$

Appendix 800

Two cars enter the Florida Turnpike at Commercial Boulevard at 8:00 A.M., each heading for Wildwood. One car's average speed is 10 miles per hour more than the other's. The faster car arrives at Wildwood at 11:00 A.M., a half an hour before the other car. What was the average speed of each car? How far did each travel?

Appendix 800

$$v = \frac{d}{t} \Rightarrow d = vt$$

$$d_1 = d_2$$

$$v_1 t_1 = v_2 t_2$$

$$(v_2 + 10)(3) = v_2(3.5)$$

$$3v_2 + 30 = 3.5v_2$$

$$0.5v_2 = 30$$

$$v_2 = 60mph, v_1 = 70mph$$

$$d = vt$$

$$d = (70)(3) = 210mi$$

Appendix 1000

Rationalize and simplify:

$$\left(\frac{\sqrt{5}-2}{\sqrt{2}+4}\right)\left(\frac{(xy)^{\frac{1}{4}}\left(x^2y^2\right)^{\frac{1}{2}}}{\left(x^2y\right)^{\frac{3}{4}}}\right)$$

Appendix 1000

$$\left(\frac{\sqrt{5}-2}{\sqrt{2}+4}\right)\left(\frac{(xy)^{\frac{1}{4}}\left(x^2y^2\right)^{\frac{1}{2}}}{\left(x^2y\right)^{\frac{3}{4}}}\right)$$

$$=\left(\frac{-\sqrt{2}+4}{-\sqrt{2}+4}\right)\left(\frac{\sqrt{5}-2}{\sqrt{2}+4}\right)\left(\frac{(xy)^{\frac{1}{4}}\left(x^2y^2\right)^{\frac{1}{2}}}{\left(x^2y\right)^{\frac{3}{4}}}\right)$$

$$=\left(\frac{-\sqrt{10}+2\sqrt{2}+4\sqrt{5}-8}{14}\right)\left(\frac{(xy)^{\frac{1}{4}}\left(x^2y^2\right)^{\frac{1}{2}}}{\left(x^2y\right)^{\frac{3}{4}}}\right)$$

$$=\left(\frac{-\sqrt{10}+2\sqrt{2}+4\sqrt{5}-8}{14}\right)\left(\frac{x^{\frac{1}{4}}y^{\frac{1}{4}}xy}{x^{\frac{3}{2}}y^{\frac{3}{4}}}\right)$$

$$=\left(\frac{-\sqrt{10}+2\sqrt{2}+4\sqrt{5}-8}{14}\right)\left(\frac{x^{\frac{5}{4}}y^{\frac{5}{4}}}{x^{\frac{3}{2}}y^{\frac{3}{4}}}\right)$$

$$=\left(\frac{-\sqrt{10}+2\sqrt{2}+4\sqrt{5}-8}{14}\right)\left(x^{\frac{-1}{4}}y^{\frac{1}{2}}\right)$$

Chapter 1 200

Find the distance between (-4,2) and (4,8)

Section 1.1

Chapter 1 200

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$d = \sqrt{\left(4 - (-4)\right)^2 + \left(8 - 2\right)^2}$$

$$d = \sqrt{100} = 10$$

Chapter 1 400

Find the Midpoint of the line connecting(-4,2) and (4,8)

Section 1.1

Chapter 1 400

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$M = \left(\frac{4 + (-4)}{2}, \frac{8 + 2}{2} \right)$$

$$M = (0, 5)$$

Chapter 1 600

Find any intercepts and axes of symmetry

$$y^2 = x + 4$$

Section 1.2

Chapter 1 600

Intercepts:

$$0 = x + 4$$

$$x = -4 \Rightarrow (-4, 0)$$

$$y^2 = 0 + 4$$

$$y = \pm 2 \Rightarrow (0, 2) \,\&\, (0, -2)$$

Axis of symmetry:

x :

$$(-y)^2 = x + 4$$

$y^2 = x + 4$ Yes, symmetric about x axis

y:

$$y^2 = (-x) + 4$$

$y^2 = -x + 4$ No, not symmetric about x axis, hence not symmetric about origin

Chapter 1 800

With the given point and slope, find the equation of the line in *slope-intercept form*.

$$P = (2, 4), m = -\frac{3}{4}$$

Section 1.3

Chapter 1 800

$$P = (2, 4) = (x_1, y_1); m = -\frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \left(-\frac{3}{4}\right)(x - 2)$$

$$y = -\frac{3}{4}x + \frac{11}{2}$$

Chapter 1 1000

Find the standard form of the equation of a circle with endpoints of a diameter at (4,3) and (0,1).

Section 1.4

Chapter 1 1000

$$diameter = d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} = \sqrt{20}$$

$$radius = r = \frac{d}{2} = \frac{\sqrt{20}}{2}$$

$$center = (h, k) = M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (2, 2)$$

then the standard form of a circle is:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-2)^2 = \left(\frac{\sqrt{20}}{2}\right)^2$$

$$(x-2)^2 + (y-2)^2 = 5$$

Chapter 2 200

If

$$f(x) = \frac{1}{x+2}, g(x) = \frac{3x}{x+3}$$

Find the domain of f(x)*g(x)

Section 2.1

Chapter 2 200

$$f(x)=\frac{1}{x+2},g(x)=\frac{3x}{x+3}$$

$$f(x)*g(x)=\frac{3x}{(x+2)(x+3)}$$

$$f(x)*g(x):\{x\mid x\in\mathbb{R},x\neq -2,-3\}$$

Chapter 2 400

Determine if the function is even, odd, or neither *algebraically.*

$$y = x^3 - 5x^2 + 2$$

Section 2.3

Chapter 2 400

To determine algebraically, substitute (-x) in for x:

$$y = x^3 - 5x^2 + 2$$

$$y = (-x)^3 - 5(-x)^2 + 2$$

$$y = -x^3 - 5x^2 + 2$$

As some signs change, but not all, we cannot conclude that it is even or odd. (Even=no signs change, Odd=all signs change) Hence it is neither.

Chapter 2 600

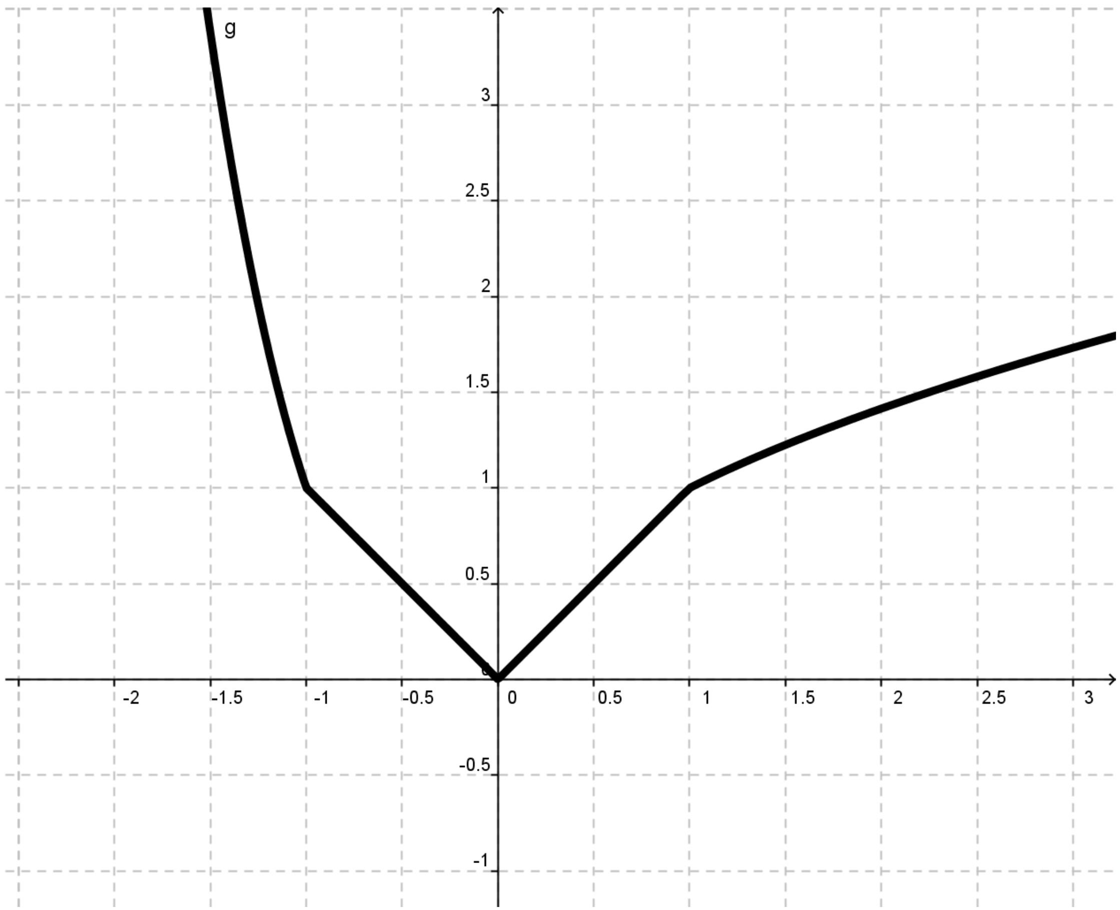
Locate all intercepts and graph the piecewise function

$$f(x) = \begin{cases} -x^3 & \text{for } x \le -1 \\ |x| & \text{for } -1 < x \le 1 \\ \sqrt{x} & \text{for } 1 < x < 9 \end{cases}$$

Section 2.4

Chapter 2 600

Only intercept in the intervals is (0,0).



Chapter 2 800

List the transformation and graph each transformation, beginning with the standard graph

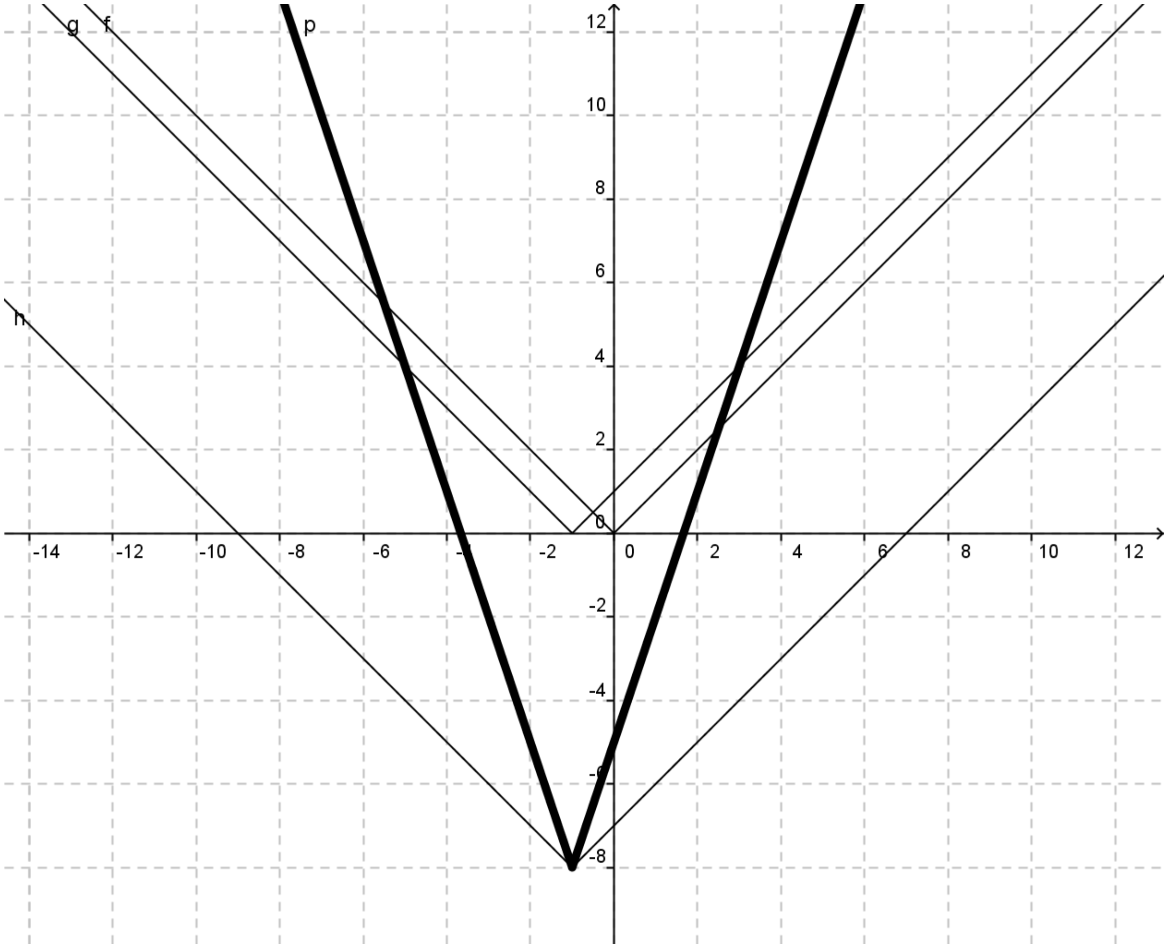
$$f(x) = 3|x+1| - 8$$

Section 2.4

Chapter 2 800

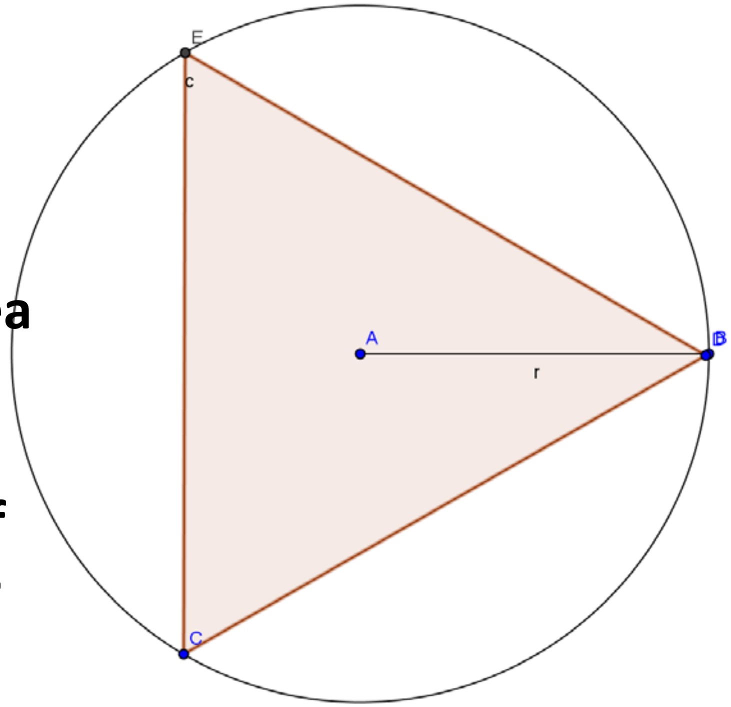
$$f(x) = 3|x+1| - 8$$

Shift one unit left
Shift eight units down
Compress by a factor of 3



Chapter 2 1000

An equilateral triangle is inscribed in a circle of radius r. Express the area within the circle, but outside the triangle as a function of the length of the triangle side, x and r



Section 2.5

Chapter 2 1000

$$A_{circle} = \pi r^2$$

$$A_{triangle} = \frac{\sqrt{3}}{4} x^2$$

(divide the equilateral triangle in half;

$base = x/2$, hypotenuse=x, find height= $\frac{\sqrt{3}}{2} x$)

$$A = A_{circle} - A_{triangle} = \pi r^2 - \frac{\sqrt{3}}{4} x^2$$

Chapter 3 200

The monthly cost *C*, in dollars, for international calls on a certain cellular phone plan is given by the function

$$C(x) = 0.38x + 5$$

Where x is the number of minutes used.

(a) What is the cost if you talk on the phone for 50 minutes?

(b) Suppose that you budgeted yourself $60 per month for the phone. What is the maximum number of (whole) minutes that you can talk?

Section 3.1

Chapter 3 200

$$C(x) = 0.38x + 5$$

$$a)$$

$$C(50) = 24$$

$$b)$$

$$60 = 0.38x + 5$$

$$x = 144\text{min}$$

Chapter 3 400

Determine the slope, *y*-intercept, where the function is increasing and decreasing and graph the function:

$$4y + 10 = 52x - 22$$

Section 3.1

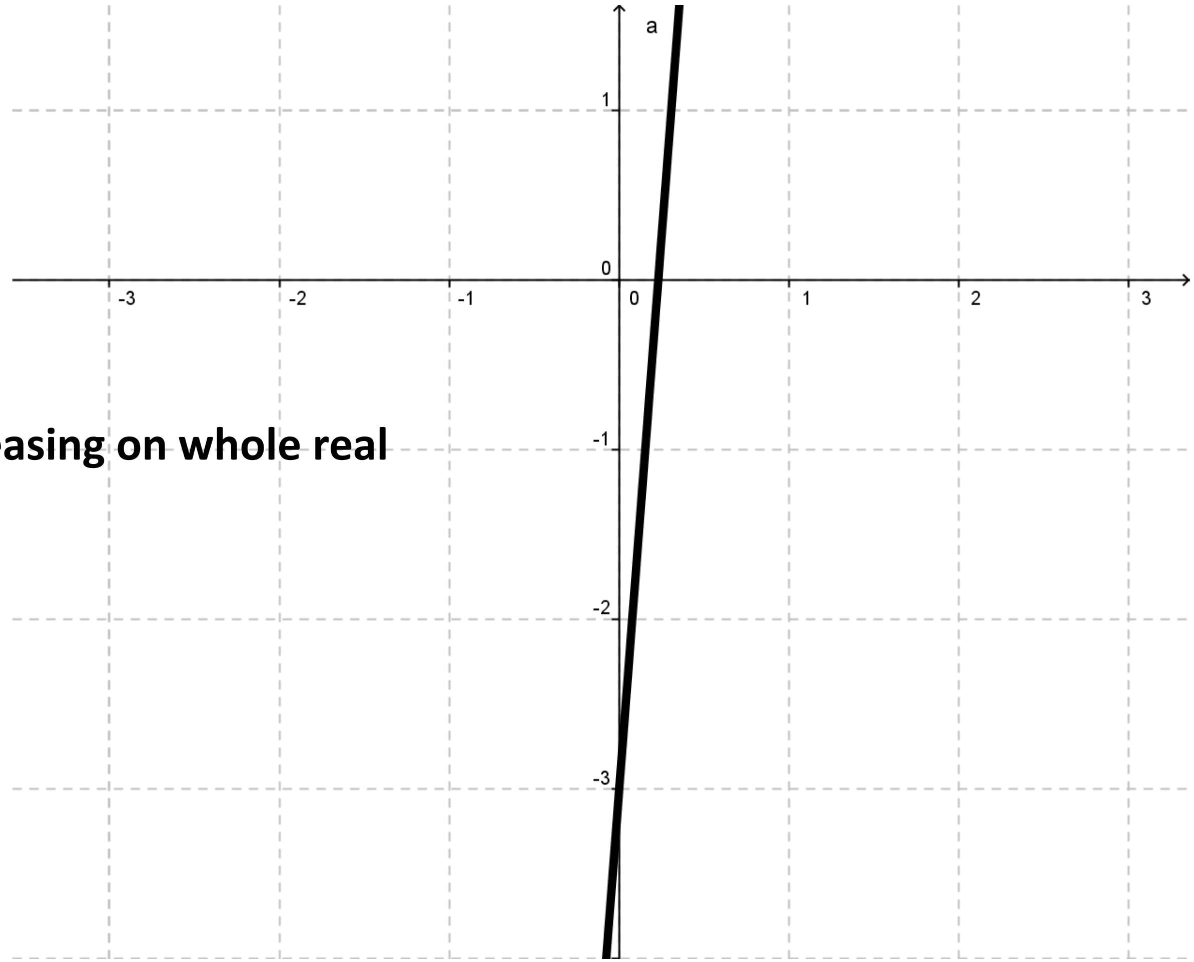
Chapter 3 400

$$4y+10=52x-22$$

$$4y=52x-12$$

$$y=13x-3$$

Increasing on whole real line



$$0=13x-3$$

$$x=\frac{3}{13}\Rightarrow\left(\frac{3}{13},0\right)$$

$$y=0-3$$

$$y=-3\Rightarrow(0,-3)$$

Chapter 3 600

Graph the function by starting with a basic parabola and use transformations. Find all intercepts and axis of symmetry. Write in $y = a(x-h)^2 + k$ **if necessary:**

$$f(x) = 3x^2 - 24x + 45$$

Section 3.3

Chapter 3 600

$$f(x) = 3x^2 - 24x + 45$$

$$f(x) = 3(x^2 - 8x + 15) = 3(x^2 - 8x + 16 - 1)$$

$$f(x) = 3(x-4)^2 - 3$$

Intercepts:

$$0 = 3(x-4)^2 - 3$$

$$\pm 1 = x - 4$$

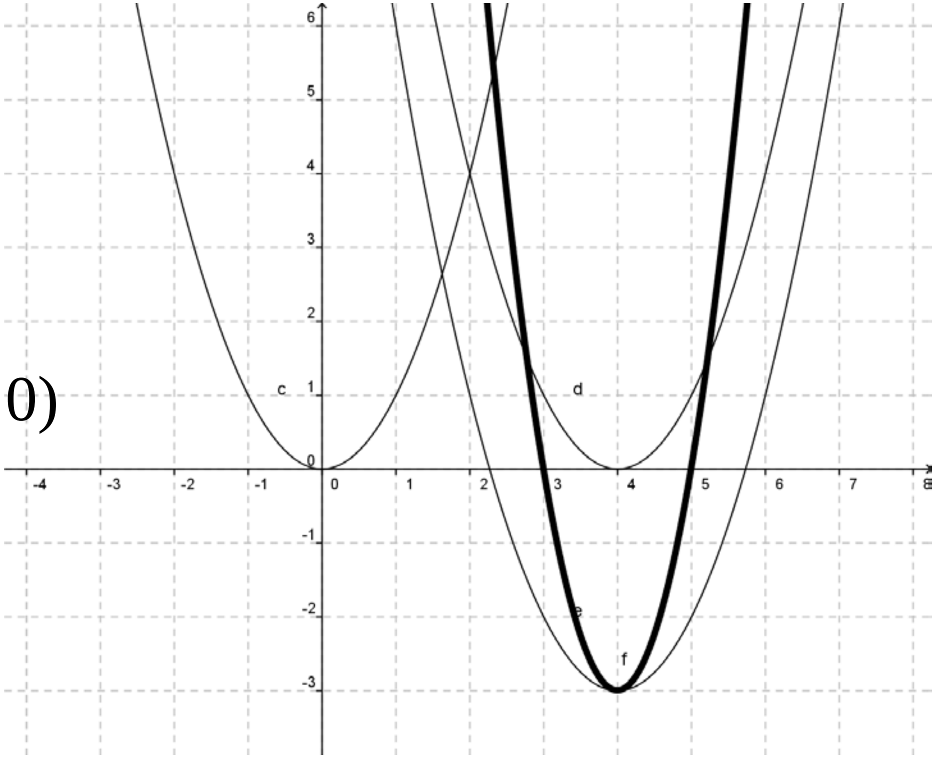
$$x = 5, 3 \Rightarrow (3,0) \,\&\, (5,0)$$

$$y = 3(0-4)^2 - 3$$

$$y = 45 \Rightarrow (0, 45)$$

Axis of Symmetry:

$$x = -\frac{b}{2a} = -\frac{-24}{6} = 4$$

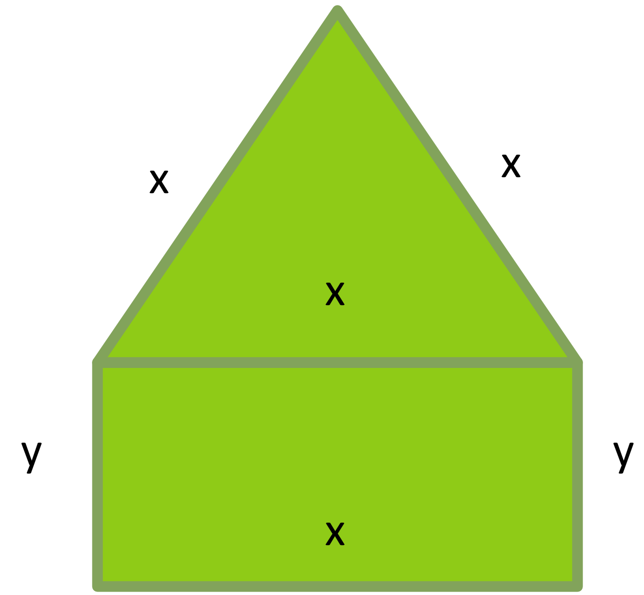


Chapter 3 800

A special window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 16 feet, what dimensions will admit the most light?

$$A_{eq.triangle} = \frac{\sqrt{3}}{4} s^2$$

Section 3.4



Chapter 3 800

$$P = 3x + 2y = 16$$

$$y = 8 - \frac{3}{2}x$$

$$A = xy + \frac{\sqrt{3}}{4}x^2$$

$$A = 8x + x^2\left(\frac{\sqrt{3}}{4} - \frac{3}{2}\right) = 8x + \left(\frac{\sqrt{3}-6}{4}\right)x^2$$

Maximum obtained at vertex of parabola:

$$x = -\frac{b}{2a} = \frac{-8}{\frac{\sqrt{3}-6}{2}} = \frac{-16}{\sqrt{3}-6} \approx 3.7\ ft$$

$$y \approx 2.5\ ft$$

$$\text{total height} = y + \frac{\sqrt{3}}{2}x \approx 5.7\ ft$$

Chapter 3 1000

Solve the inequality

$$25x^2 + 16 < 40x$$

Section 3.5

Chapter 3 1000

$$25x^2 + 16 < 40x$$

$$25x^2 - 40x + 16 < 0$$

$$x^2 - \frac{8}{5}x + \frac{16}{25} < 0$$

$$\left(x - \frac{4}{5}\right)^2 < 0$$

$$x - \frac{4}{5} < 0, x - \frac{4}{5} > 0$$

$$x < \frac{4}{5}, x > \frac{4}{5} \Rightarrow x = \frac{4}{5}$$

$\therefore$ we can conclude that the graph is nonnegative meaning there are no values less than 0

Chapter 4 200

Find the intercepts, where the function touches or crosses the x-axis, the number of turning points, and determine the end behavior of the function. Sketch the function.

$$h(x) = x(x+2)(x+4)$$

Section 4.1

Chapter 4 200

Intercepts:

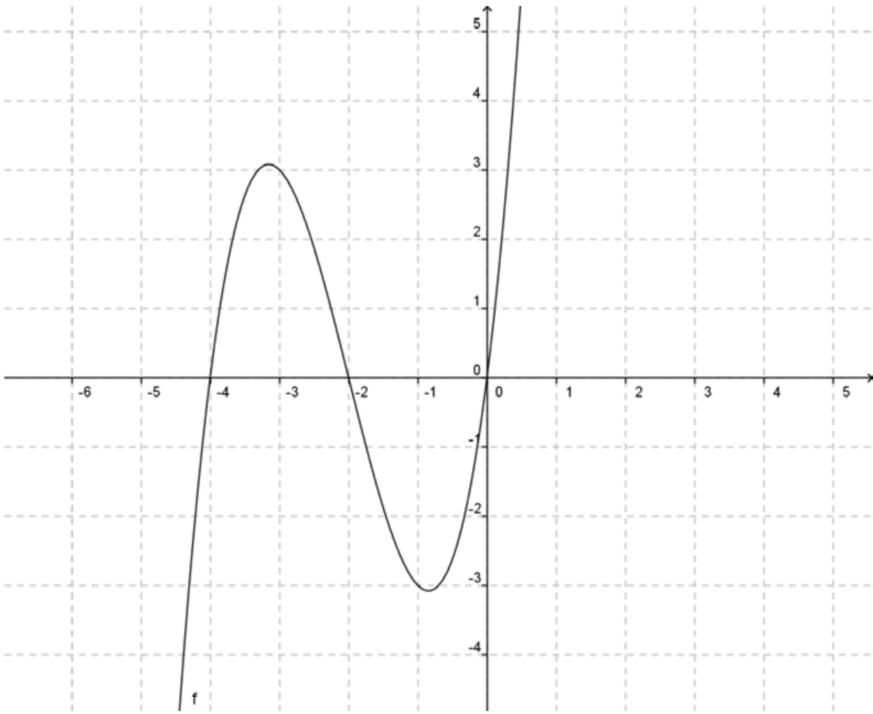
x: x=0, x=-2, x=-4

y: y=0

Crosses at all x intercepts due to odd multiplicity

Number of Turning Points: 2

For x>>0, f(x) goes to infinity, for x<<0, f(x) goes to negative infinity



Chapter 4 400

Find the domain and any horizontal, vertical, and oblique asymptotes

$$G(x) = \frac{x^3 - 1}{x - x^2}$$

Section 4.2

Chapter 4 400

$$G(x)=\frac{x^3-1}{x-x^2}=\frac{(x-1)(x^2+x-1)}{x(1-x)}=\frac{(x-1)(x^2+x-1)}{-x(x-1)}=-\frac{(x^2+x-1)}{x}$$

Domain: All reals except x=0,x=1, hole at x=1
VA: x=0
HA: none since (degree numerator)>(degree denominator)
OA: y=-x-1 after long division

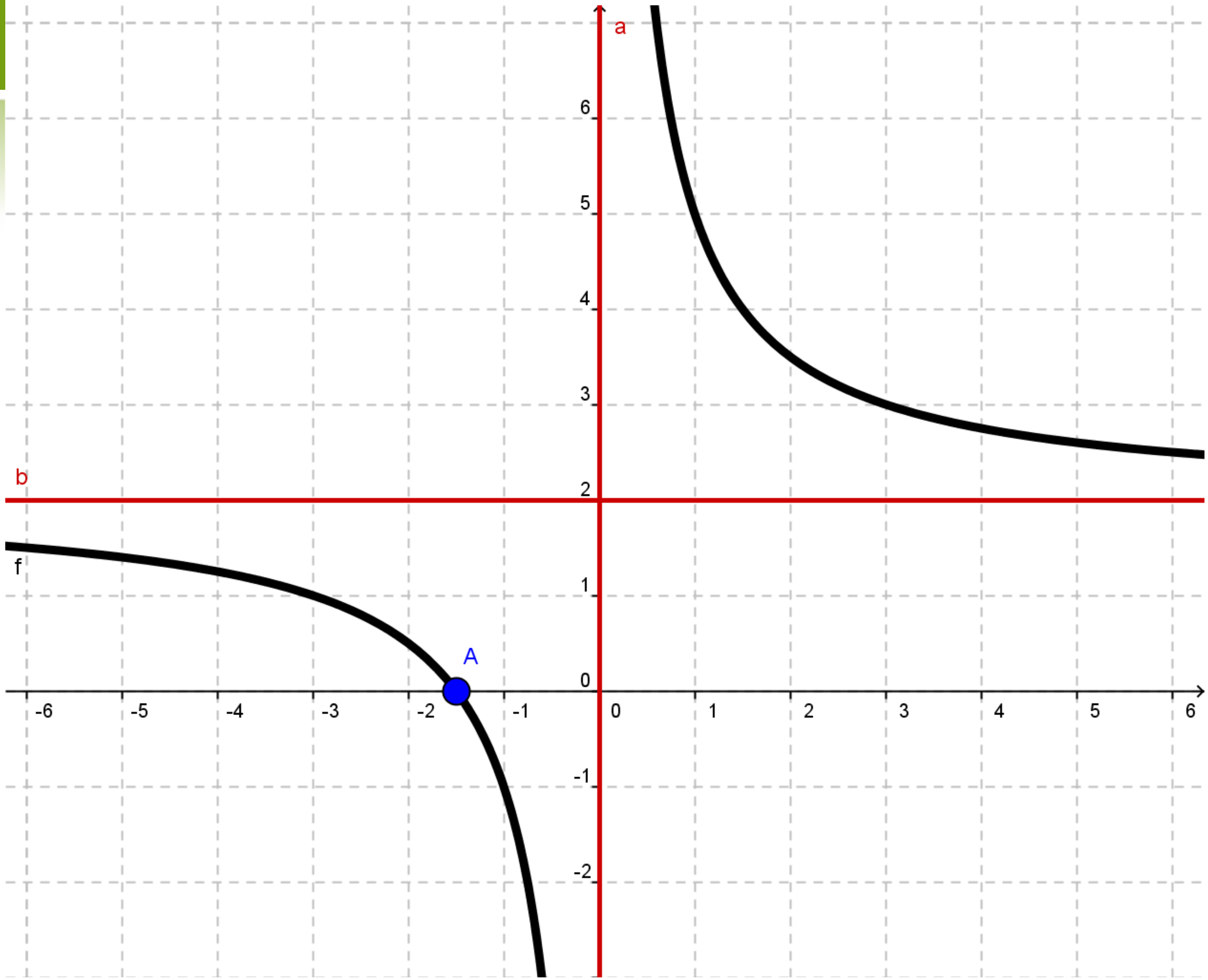
CHAPTER 4 600

Go through the seven step process to obtain the graph of the function:

$$R(x) = \frac{2x^2 - 7x - 15}{x^2 - 5x}$$

Section 4.3

Chapter 4 600



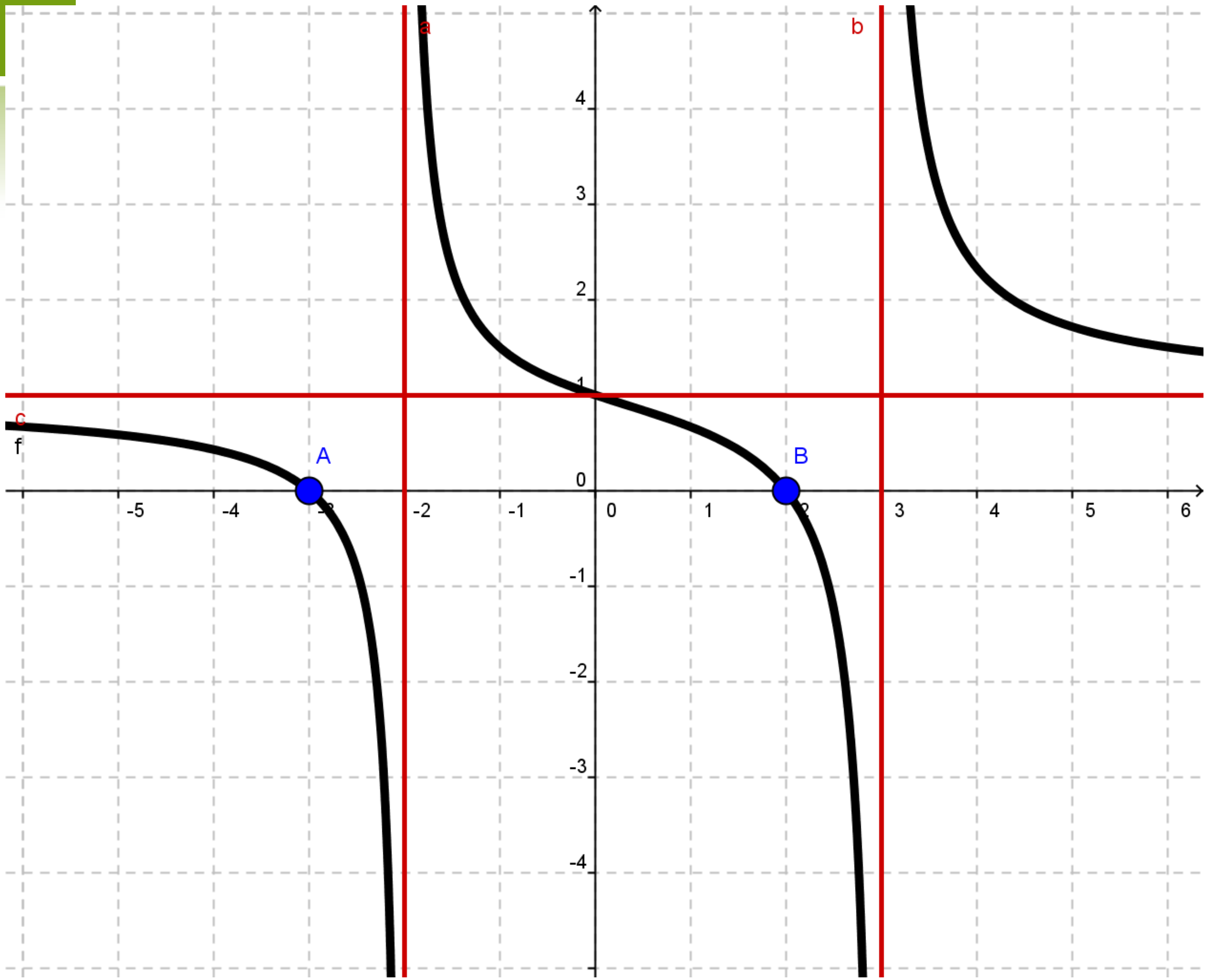
Chapter 4 800

Go through the seven step process to obtain the graph of the function:

$$R(x) = \frac{x^3}{x^2 - 4}$$

Section 4.3

Chapter 4 800



Chapter 4 1000

Solve & Graph the solution set.

$$\frac{(x-2)(x-1)}{x-3} \geq 0$$

Section 4.4

Chapter 4 1000

$$\frac{(x-2)(x-1)}{x-3} \geq 0$$

Critical pts: 3, 2, 1

Intervals:

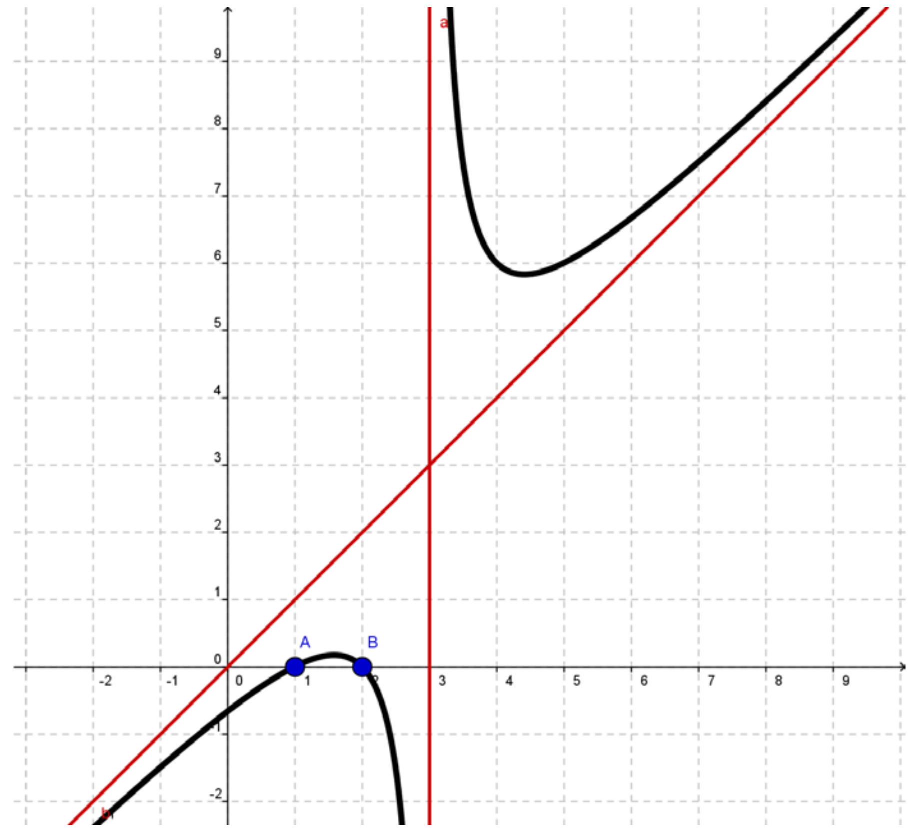
$$(-\infty,1) \Rightarrow 0 \Rightarrow \leq 0$$

$$(1,2) \Rightarrow 1.5 \Rightarrow \geq 0$$

$$(2,3) \Rightarrow 2.5 \Rightarrow \leq 0$$

$$(3,\infty) \Rightarrow 4 \Rightarrow \geq 0$$

$$\therefore (1,2) \cup (3,\infty)$$



Chapter 5 200

Find

$$f \circ g(x) \text{ and } g \circ f(x)$$

$$f(x) = \frac{x}{x+3}; \quad g(x) = \frac{2}{x}$$

Section 5.1

Chapter 5 200

$$f(x)=\frac{x}{x+3};\quad g(x)=\frac{2}{x}$$

$$f\circ g(x)=\frac{\frac{2}{x}}{\frac{2}{x}+3}=\frac{2}{x\left(\frac{2+3x}{x}\right)}=\frac{2}{2+3x}$$

$$g\circ f(x)=\frac{2}{\frac{x}{x+3}}=\frac{2(x+3)}{x}=2+\frac{6}{x}$$

Chapter 5 400

Find the inverse of the function.

$$g\left(x\right)=\frac{-3x-4}{x-2}$$

Section 5.2

Chapter 5 400

$$g(x) = y = \frac{-3x-4}{x-2}$$

$$x = \frac{-3y-4}{y-2}$$

$$xy - 2x = -3y - 4$$

$$xy + 3y = 2x - 4$$

$$y(x+3) = 2x - 4$$

$$y^{-1} = g^{-1}(x) = \frac{2x-4}{x+3}$$

Chapter 5 600

Solve the equation. Express any irrational solutions in exact form.

$$2 \cdot 49^{x} + 11 \cdot 7^{x} + 5 = 0$$

Sections 5.3 & 5.6

Chapter 5 600

$$2 \cdot 49^x + 11 \cdot 7^x + 5 = 0$$

$$2 \bullet 7^{2x} + 11 \cdot 7^x + 5 = 0$$

$$\text{let } y = 7^x$$

$$2y^2 + 11y + 5 = 0$$

$$(2y + 1)(y + 5) = 0$$

$$(2 \cdot 7^x + 1)(7^x + 5) = 0$$

$$2 \cdot 7^x + 1 = 0 \quad \text{or} \quad 7^x + 5 = 0$$

$$7^x = -\frac{1}{2} \quad \text{or} \quad 7^x = -5$$

$$x \ln 7 = \ln -\frac{1}{2} \quad \text{or} \quad x \ln 7 = \ln -5$$

$$x = \frac{\ln\left(-\frac{1}{2}\right)}{\ln 7} \quad \text{or} \quad x = \frac{\ln(-5)}{\ln 7}$$

no real solution

Chapter 5 800

Write the expression as a single logarithm

$$\log\left(\frac{x^2+2x-3}{x^2-4}\right)-\log\left(\frac{x^2+7x+6}{x+2}\right)$$

Section 5.5

Chapter 5 800

$$\log\left(\frac{x^2+2x-3}{x^2-4}\right)-\log\left(\frac{x^2+7x+6}{x+2}\right)$$

$$=\log\left(\frac{(x-2)(x-1)}{(x-2)(x+2)}\right)-\log\left(\frac{(x+6)(x+1)}{x+2}\right)$$

$$=\log\left(\frac{(x-1)}{(x+2)}\bullet\frac{(x+2)}{(x+6)}\right)=\log\left(\frac{x-1}{x+6}\right)$$

Chapter 5 1000

What will a $90,000 house cost 5 years from now if the price appreciation for homes over that period averages 3% compounded annually?

Section 5.7

Chapter 5 1000

$P = 90,000$

$t = 5$

$r = 0.03$

$n = 1$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A = 90,000(1.03)^5$

$A \approx 104,334.67$

Approximately 104,334.67 dollars